

Symplectic Geometry

Homework 6

Exercise 1. (6 points)

Let X_f be the Hamiltonian vector field associated to a function $f \in C^\infty(X)$. Prove that X_f is tangent to the level sets $\{f = \text{const}\}$.

Exercise 2. (28 points)

Exercises 1 (7 points), 2 (7 points), 3 (7 points) and 4 (7 points) from Homework 3 (on page 20) in *Lectures on Symplectic Geometry* by A. Cannas da Silva. Also available online at: <http://www.mi.uni-koeln.de/~pabiniak/sg.html>

Note that the flow of v denoted by Cannas da Silva $\exp(tv)$ in the lecture was usually denoted by ψ_t , and that the lift of a diffeomorphism f to the cotangent bundle was denoted in lecture by \tilde{f} whereas in the book by Cannas da Silva it is denoted by $f_\#$.

Exercise 3. (6 points)

Let X be compact, connected, and not just a point, and let $f \in C^\infty(X)$. Find the number of intersection points between $X_{df} = \text{graph}(df)$ and X_0 in T^*X , and show that it is at least 2.